

第六章 频率响应

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放大电路的频率响应

研究放大电路的动态指标（主要是增益）随信号频率变化时的响应。

6.2 单时间常数RC电路的频率响应

6.4.3 BJT的高频小信号模型及频率参数

6.4.4 单级共射极放大电路的频率响应

6.2 单时间常数RC电路的频率响应

1. RC低通电路的频率响应

①增益频率函数（电路理论中的稳态分析）

RC电路的电压增益（传递函数）：

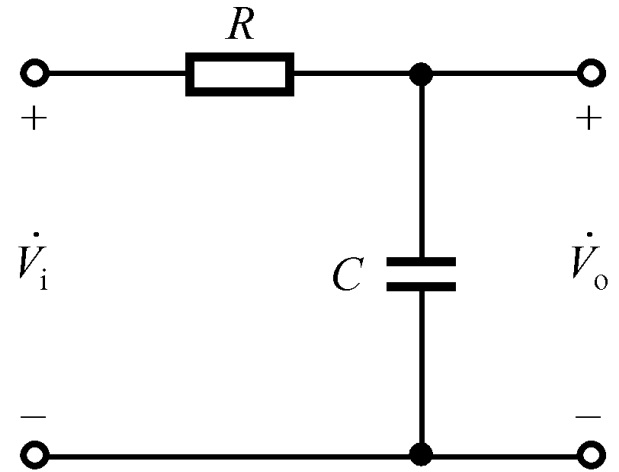
$$A_{\text{vH}}(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/sC_1}{R_1 + 1/sC_1} = \frac{1}{1 + sR_1C_1}$$

又 $s = j\omega = j2\pi f$ 且令 $f_H = \frac{1}{2\pi R_1C_1}$

则
$$\dot{A}_{\text{vH}} = \frac{\dot{V}_o}{\dot{V}_i} = \frac{1}{1 + j(f/f_H)}$$

电压增益的幅值（模）
$$A_{\text{vH}} = \frac{1}{\sqrt{1 + (f/f_H)^2}} \quad (\text{幅频响应})$$

电压增益的相角
$$\varphi_H = -\arctan(f/f_H) \quad (\text{相频响应})$$



RC低通电路

1. RC低通电路的频率响应

② 频率响应曲线描述

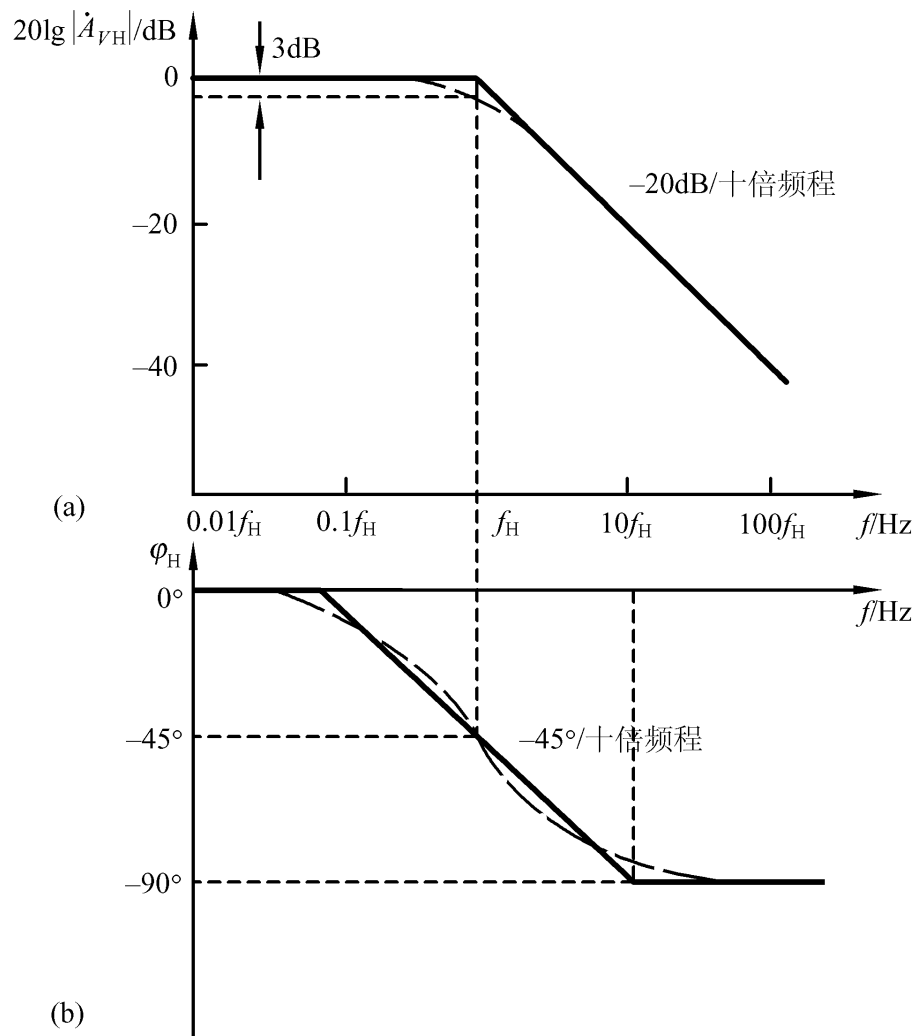
幅频响应

$$A_{VH} = \frac{1}{\sqrt{1 + (f / f_H)^2}}$$

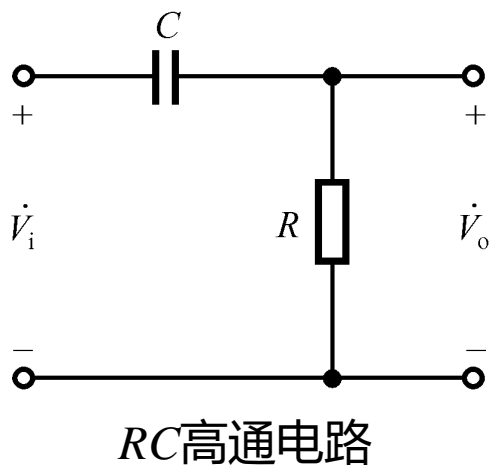
最大误差 -3dB

相频响应

$$\varphi_H = -\arctan(f / f_H)$$



2. RC高通电路的频率响应



RC电路的电压增益:

$$A_{VL}(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + 1/sC_2}$$
$$= \frac{s}{s + 1/R_2C_2}$$

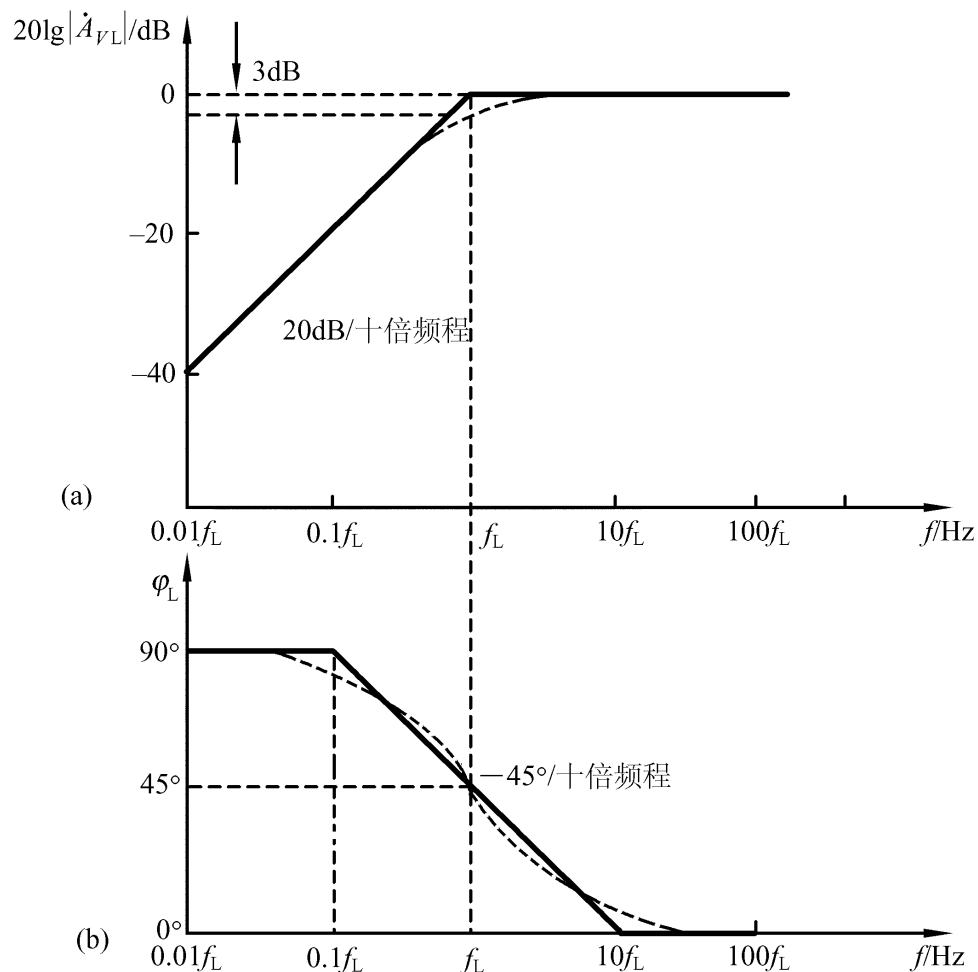
幅频响应

$$A_{VL} = \frac{1}{\sqrt{1 + (f_L / f)^2}}$$

相频响应

$$\varphi_L = \arctan(f_L / f)$$

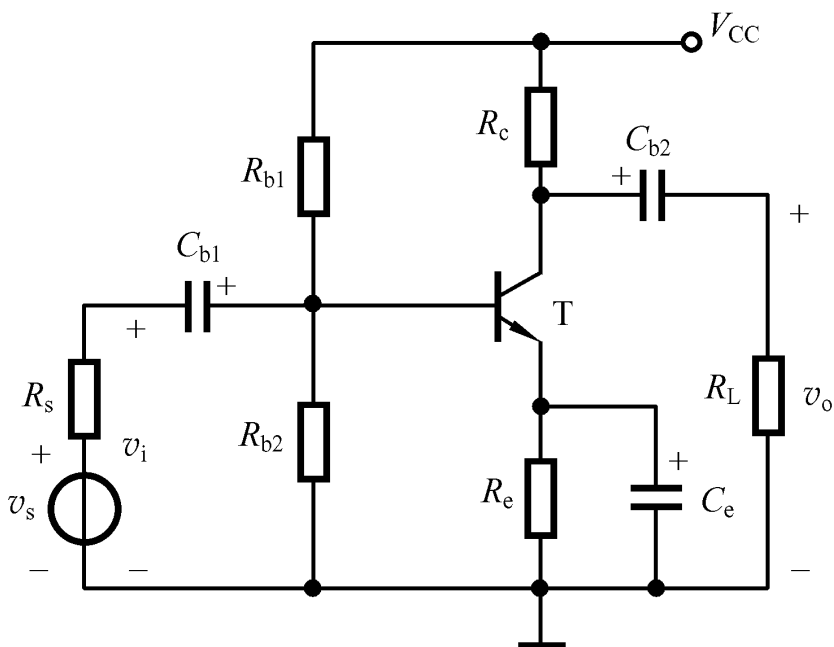
输出超前输入



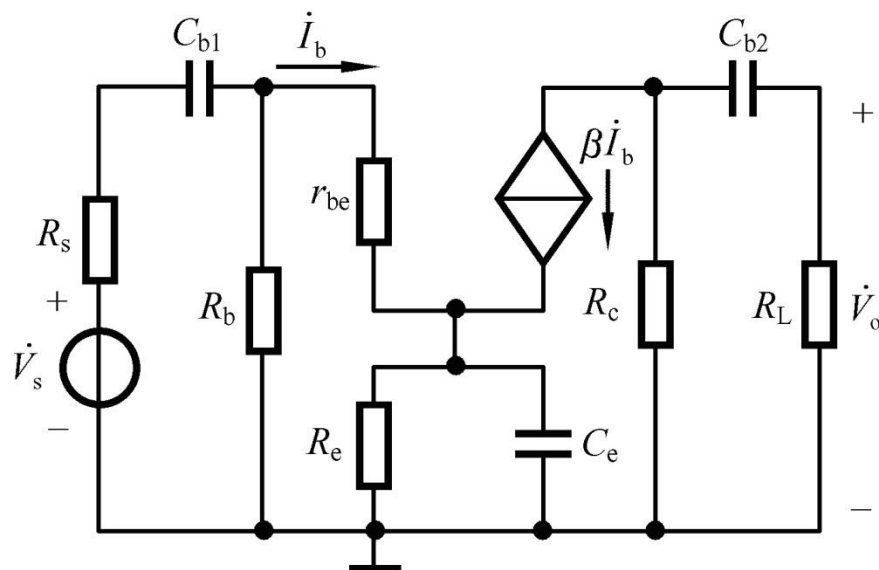
4.7.3 单级共射极放大电路的频率响应

2. 低频响应

① 低频等效电路



(a)



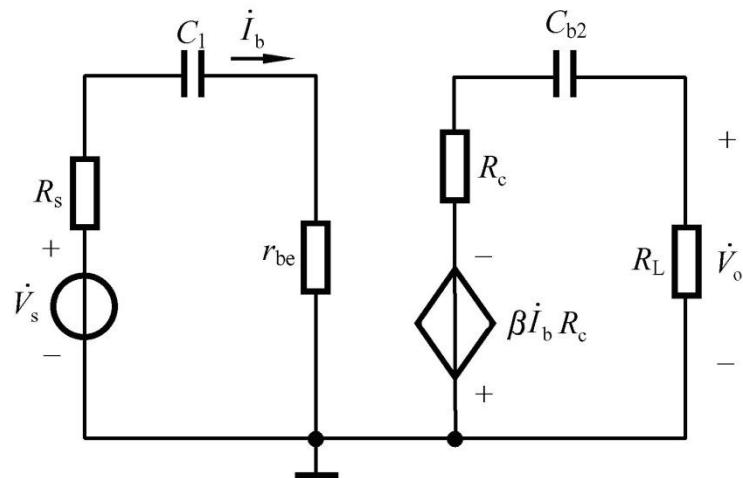
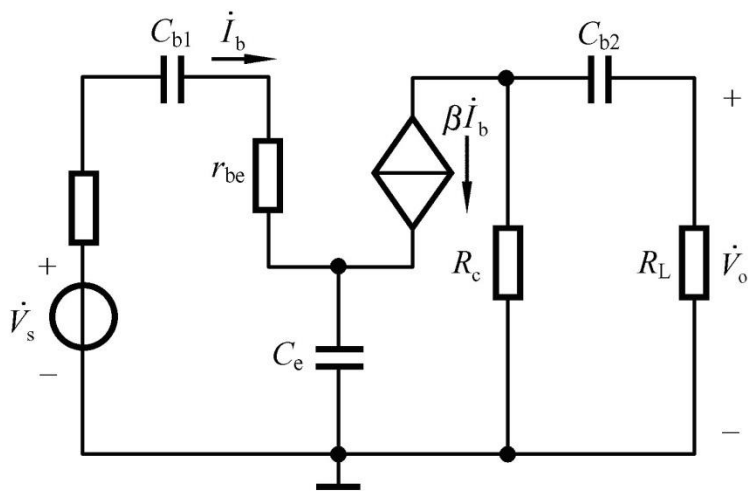
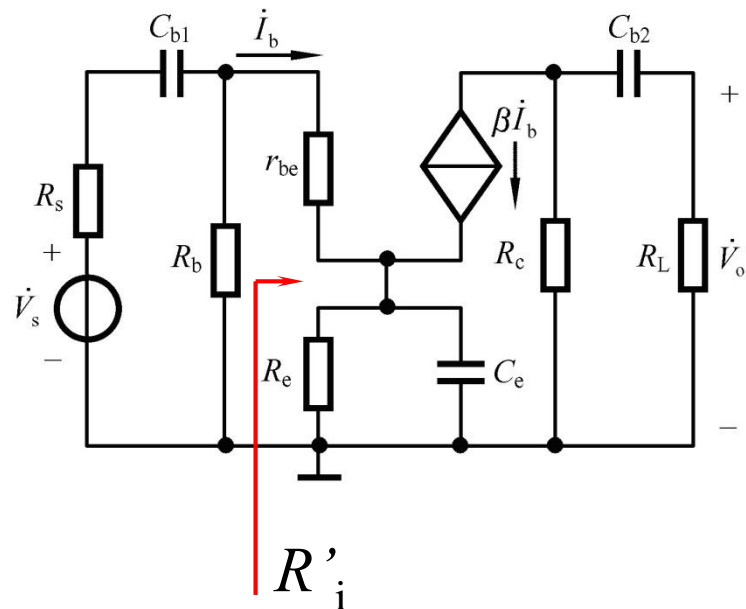
2. 低频响应

① 低频等效电路

$$R_b = (R_{b1} // R_{b2}) \text{ 远大于 } R'_i$$

$$\frac{1}{\omega C_e} \ll R_e$$

$$I_e \approx I_c, C_e \gg C_{b2}$$



2. 低频响应

② 低频响应

$$C_1 = \frac{C_{b1} C_e}{(1 + \beta) C_{b1} + C_e}$$

$$\dot{A}_{VSL} = \frac{\dot{V}_o}{\dot{V}_s} = -\frac{\beta R'_L}{R_s + r_{be}} \cdot \frac{1}{1 - j/\omega C_1 (R_s + r_{be})} \cdot \frac{1}{1 - j/\omega C_{b2} (R_c + R_L)}$$

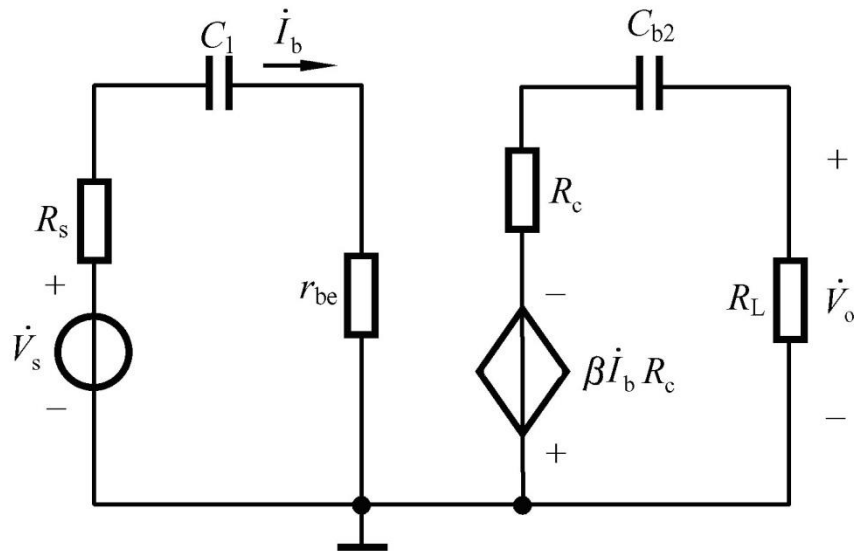
当 $\dot{A}_{VSM} = -\frac{\beta R'_L}{R_s + r_{be}}$ 中频区(即通常内)源电压增益

$$f_{L1} = \frac{1}{2\pi C_1 (R_s + r_{be})}$$

$$f_{L2} = \frac{1}{2\pi C_{b2} (R_c + R_L)}$$

则 $\dot{A}_{VSL} = \frac{\dot{A}_{VSM}}{[1 - j(f_{L1}/f)][1 - j(f_{L2}/f)]}$

当 $f_{L1} > 4f_{L2}$ 下限频率取决于 f_{L1}



2. 低频响应

② 低频响应

$$\dot{A}_{VSL} = \frac{\dot{A}_{VSM}}{[1 - j(f_{L1}/f)][1 - j(f_{L2}/f)]}$$

当 $f_{L1} > 4f_{L2}$ 时,

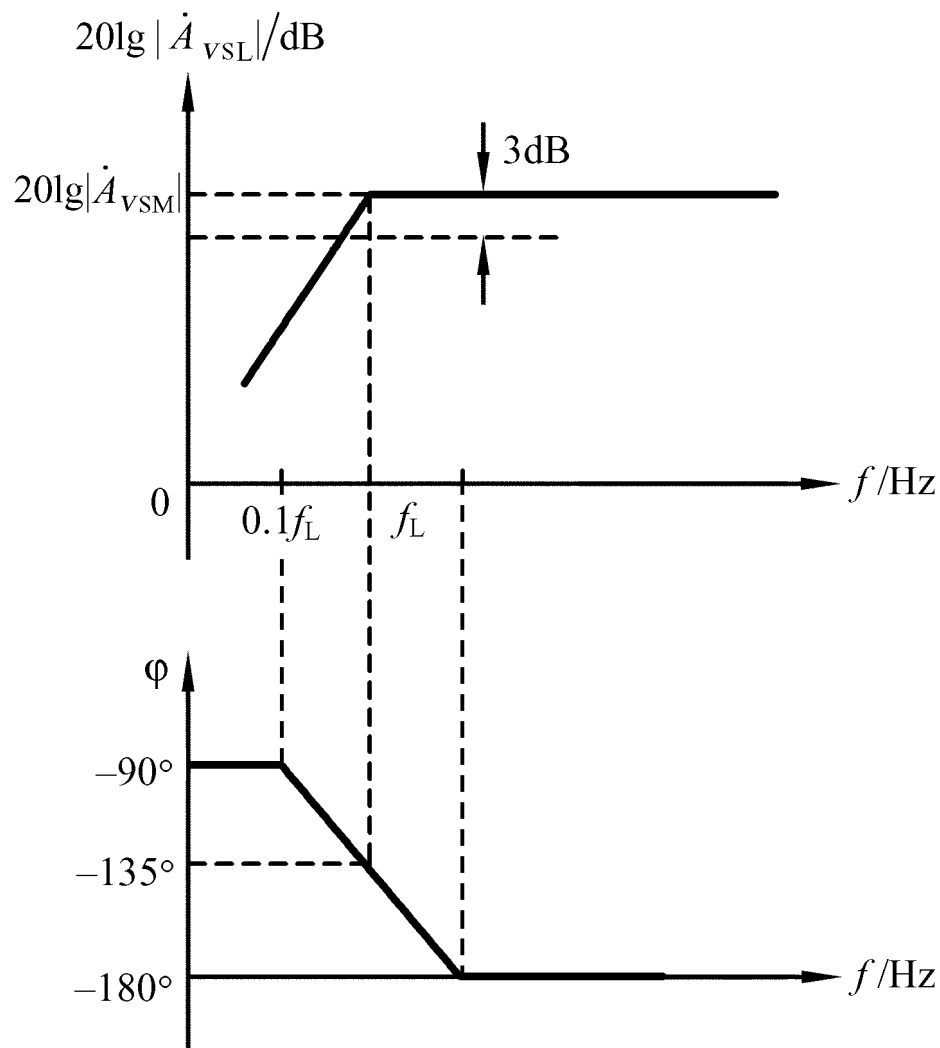
下限频率取决于 f_{L1}

$$\dot{A}_{VSL} = \dot{A}_{VSM} \cdot \frac{1}{1 - j(f_{L1}/f)}$$

幅频响应

$$20\lg|\dot{A}_{VSL}| = 20\lg|\dot{A}_{VSM}| + 20\lg \frac{1}{\sqrt{1 + (f_{L1}/f)^2}}$$

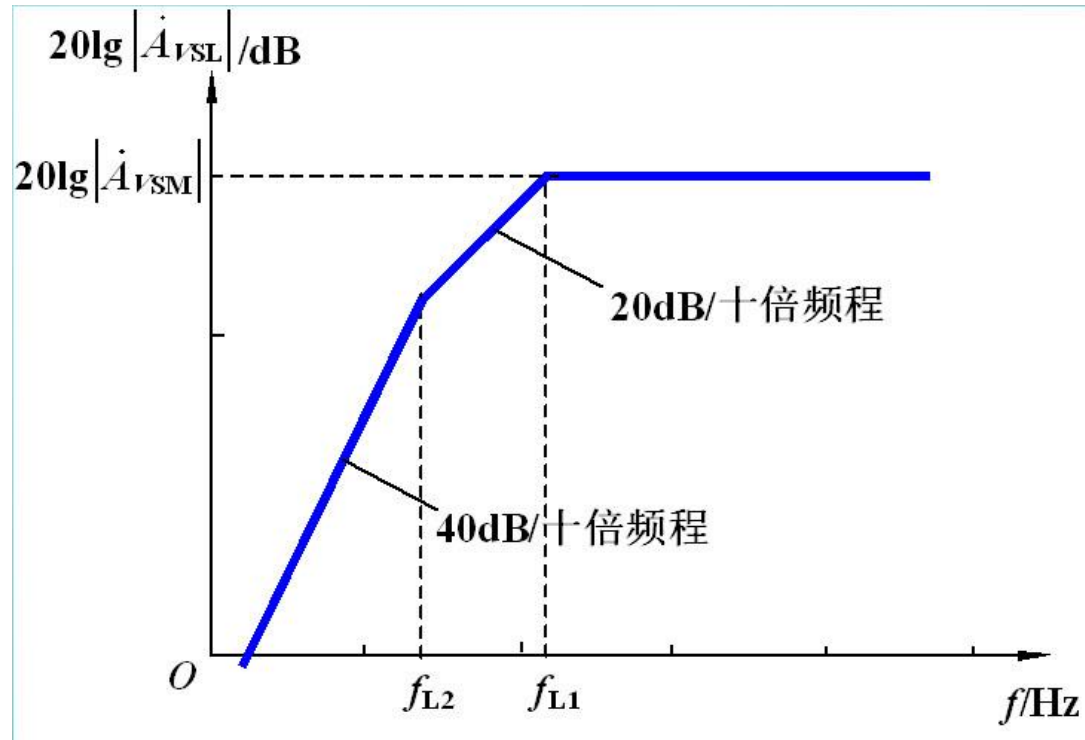
相频响应 $\varphi = -180^\circ - \arctg(-f_{L1}/f) = -180 + \arctg(f_{L1}/f)$



2. 低频响应

② 低频响应

包含 f_{L2} 的幅频响应



6.4.3 BJT的高频小信号模型及频率参数

1. BJT的高频小信号模型

①模型的引出

$r_{bb'}$ ---基区的体电阻，b'是假想的基区内的一个点

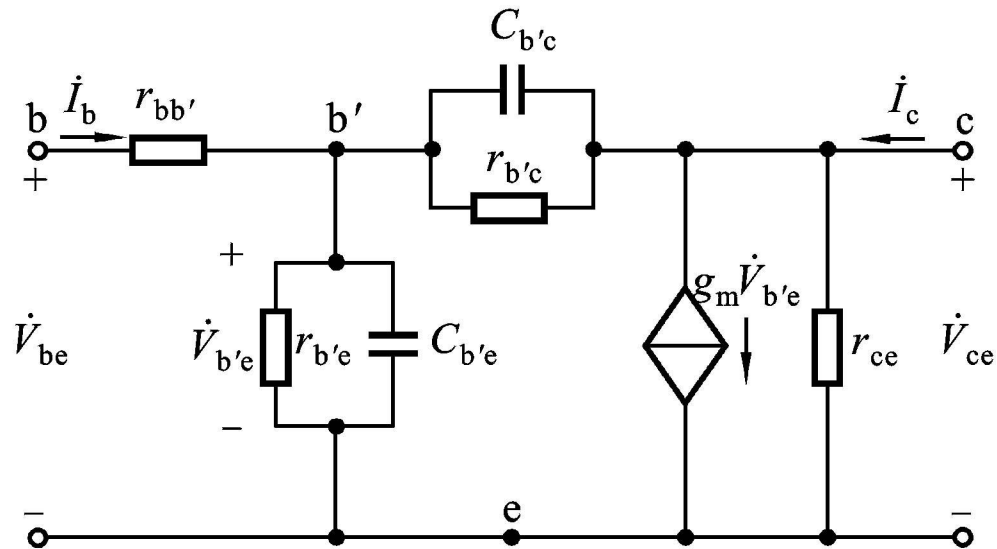
$r_{b'e}$ ---发射结电阻 r_e 归算到基极回路的电阻

$C_{b'e}$ ---发射结电容

$r_{b'c}$ ---集电结电阻

$C_{b'c}$ ---集电结电容

互导
$$g_m = \left. \frac{\partial i_C}{\partial v_{B'E}} \right|_{V_{CE}} = \left. \frac{\Delta i_C}{\Delta v_{B'E}} \right|_{V_{CE}}$$



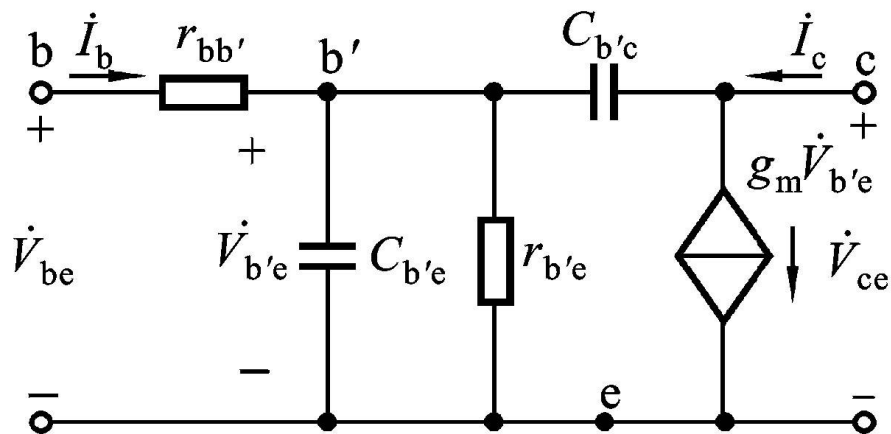
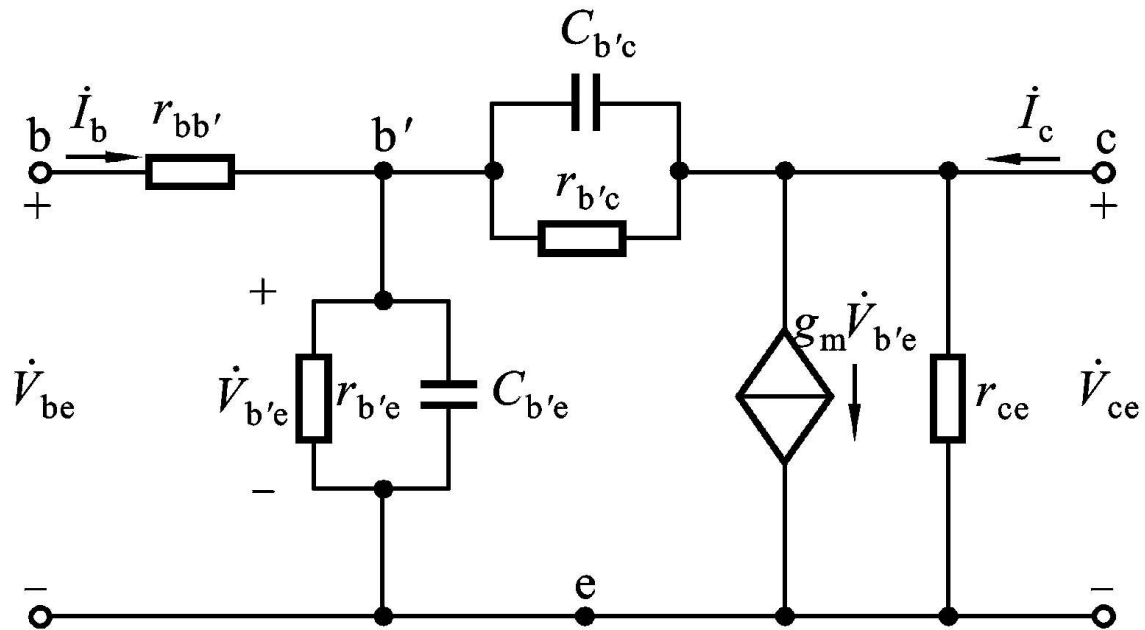
BJT的高频小信号模型

1. BJT的高频小信号模型

②简化模型

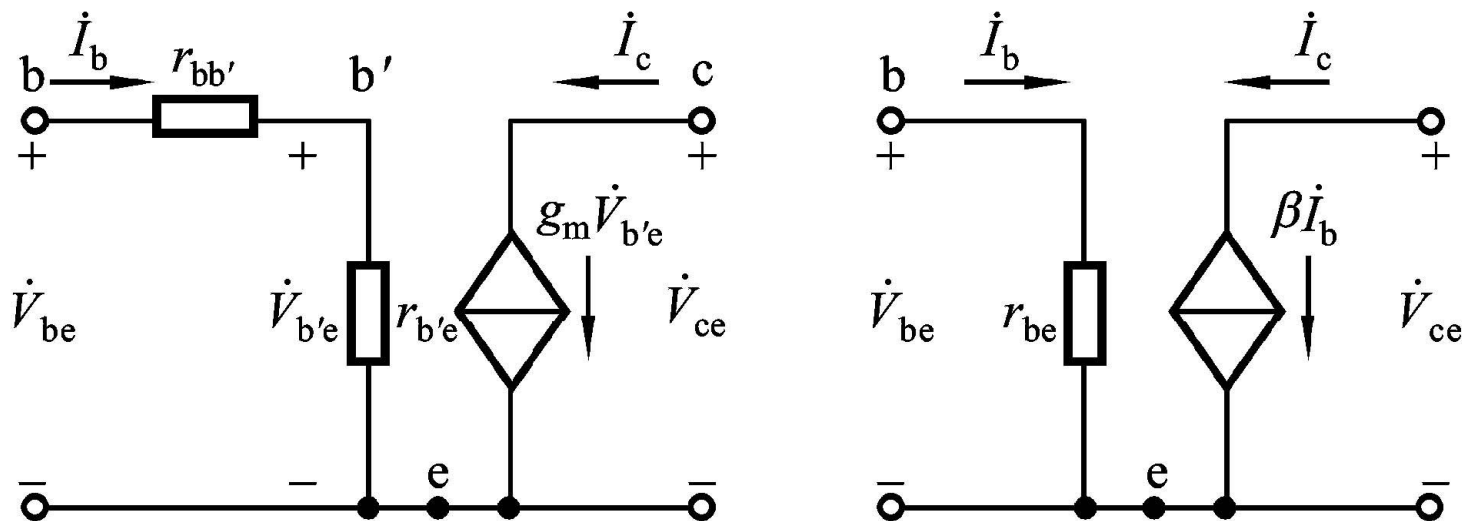
忽略 $r_{b'c}$ 和 r_{ce}

混合 Π 型高频小信号模型



2. BJT高频小信号模型中元件参数值的获得

低频时，混合 Π 模型与H参数模型等价



$$r_{be} = r_{bb'} + r_{b'e}$$

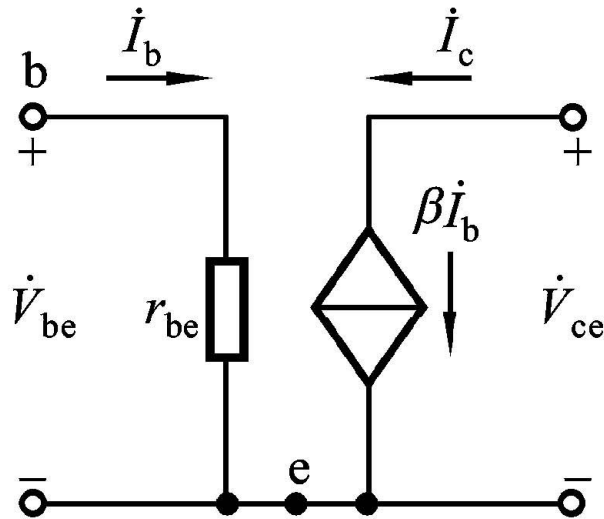
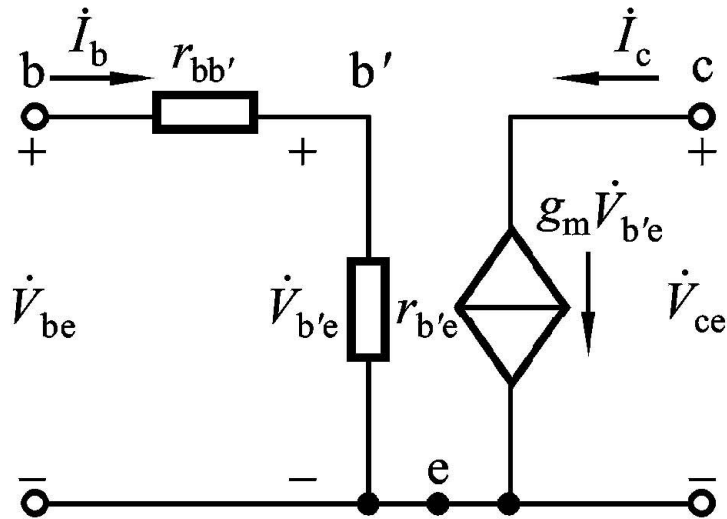
$$\text{又 } r_{be} = r_{bb'} + (1 + \beta)r_e = r_{bb'} + (1 + \beta) \frac{V_T}{I_{EQ}}$$

所以 $r_{b'e} = (1 + \beta) \frac{V_T}{I_{EQ}}$

$$r_{bb'} = r_{be} - r_{b'e}$$

2. BJT高频小信号模型中元件参数值的获得

低频时，混合 Π 模型与H参数模型等价



又因为 $\dot{V}_{b'e} = \dot{I}_b r_{b'e}$

$$g_m \dot{V}_{b'e} = \beta \dot{I}_b$$

所以 $g_m = \frac{\beta}{r_{b'e}} \approx \frac{I_{EQ}}{V_T}$

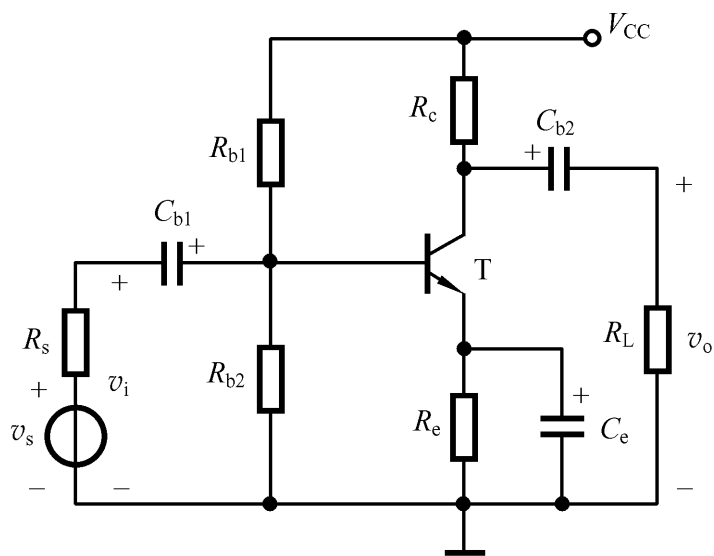
$$C_{b'e} = \frac{g_m}{2\pi f_T}$$

$C_{b'e}$ 和 f_T 从手册中查出

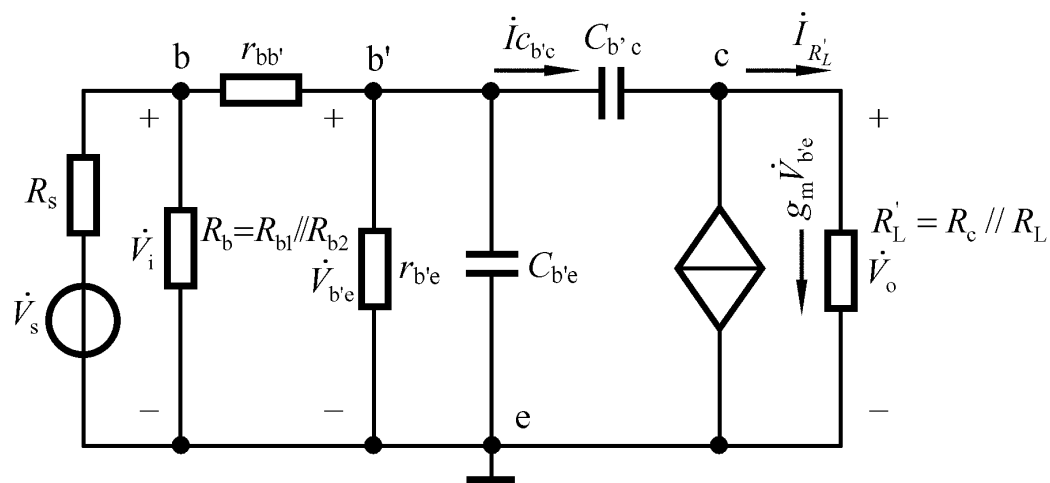
6.4.4 单级共射极放大电路的频率响应

1. 高频响应

① Π 型高频等效电路



(a)



(b)

6.4.4 单级共射极放大电路的频率响应

1. 高频响应

① Π 型高频等效电路

目标：断开输入输出之间的连接

对节点 c 列KCL得

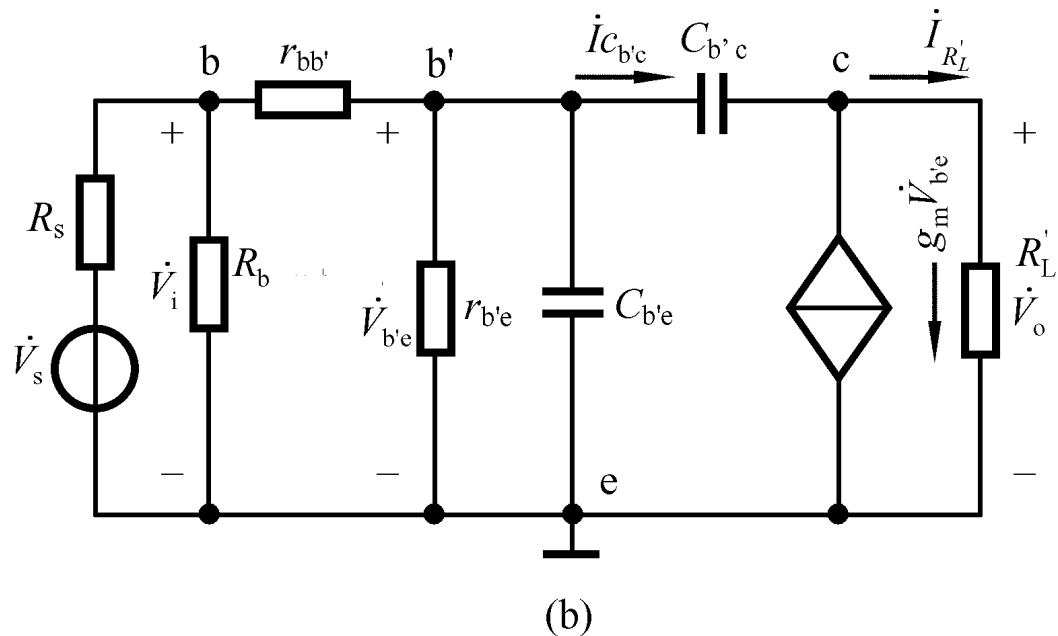
$$g_m \dot{V}_{b'e} + \frac{\dot{V}_o}{R'_L} + (\dot{V}_o - \dot{V}_{b'e})j\omega C_{b'c} = 0$$

由于输出回路电流比较大，所以可以忽略 $C_{b'c}$ 的分流，得

$$\dot{V}_o \approx -g_m R'_L \dot{V}_{b'e}$$

而输入回路电流比较小，所以不能忽略 $C_{b'c}$ 的电流。

又因为 $\dot{I}_{C_{b'c}} = (\dot{V}_{b'e} - \dot{V}_o)j\omega C_{b'c}$



$$Z_M = \frac{\dot{V}_{b'e}}{\dot{I}_{C_{b'c}}} = \frac{1}{(1 + g_m R'_L)j\omega C_{b'c}}$$

相当于 b' 和 e 之间存在一个电容

若用 C_{M1} 表示，则 $C_{M1} = (1 + g_m R'_L)C_{b'c}$

C_{M1} 称为密勒电容

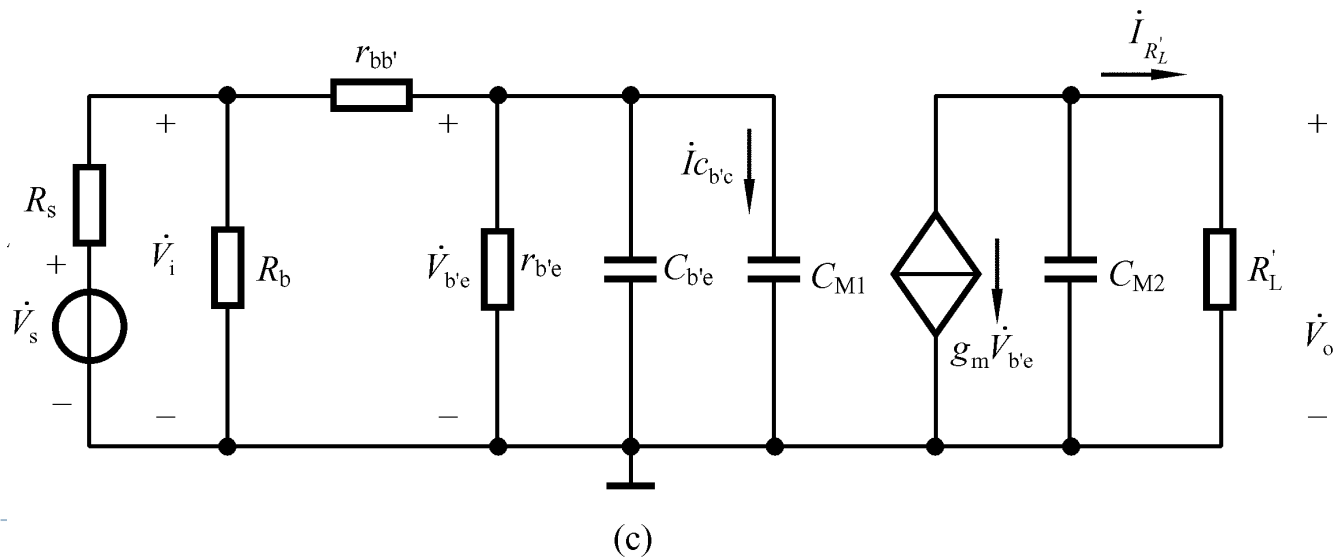
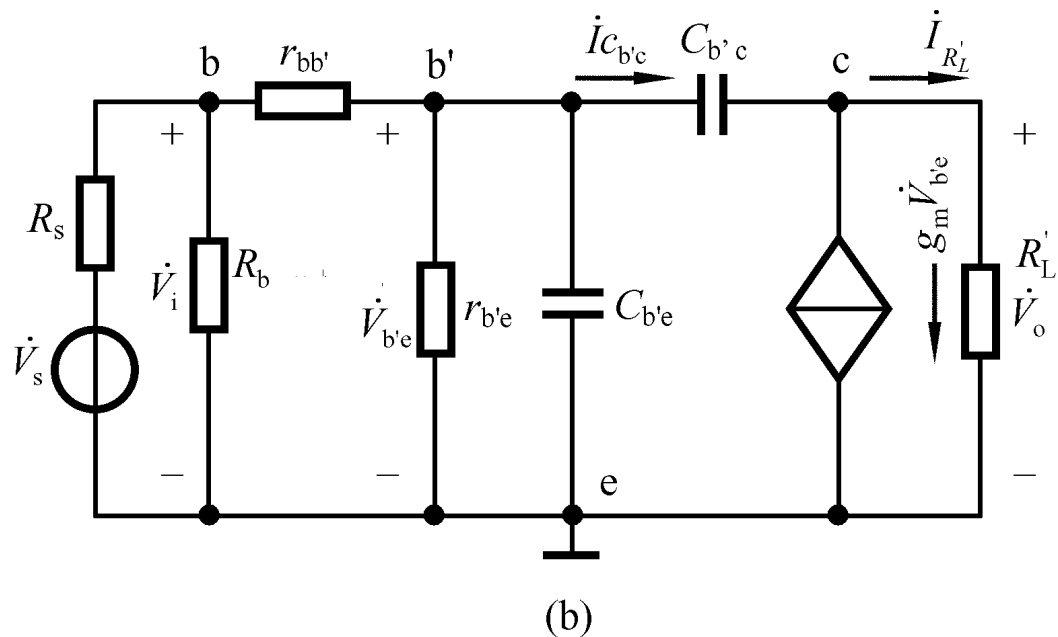
6.4.4 单级共射极放大电路的频率响应

1. 高频响应

① Π 型高频等效电路

$$C_{M1} = (1 + g_m R'_L) C_{b'c}$$

同理，在c、e之间也可以求得一个等效电容 C_{M2} ，且 $C_{M2} \approx C_{b'c}$
等效后断开了输入输出之间的联系



6.4.4 单级共射极放大电路的频率响应

1. 高频响应

① Π 型高频等效电路

目标：简化和变换

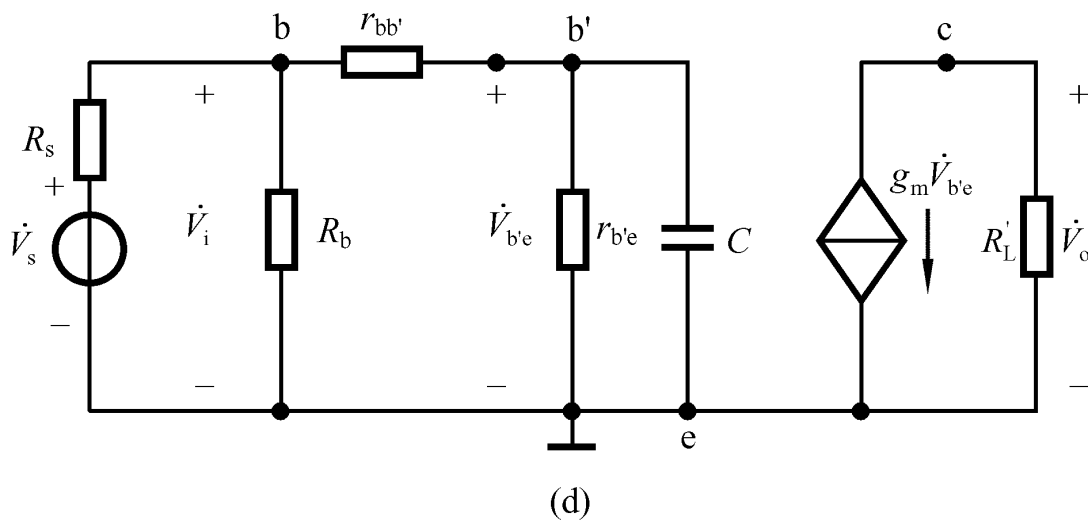
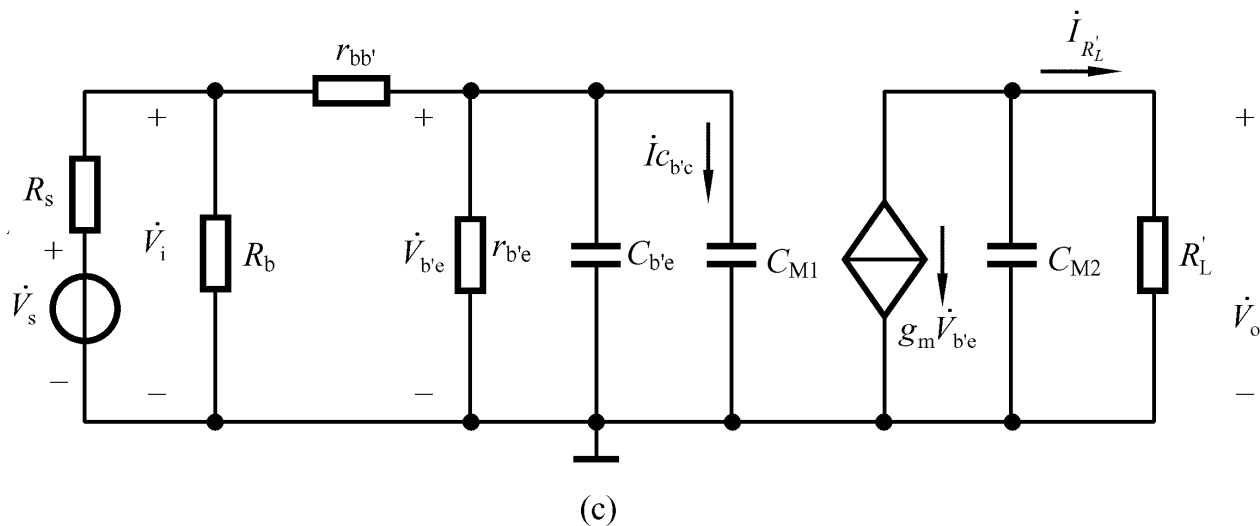
$$C_{M1} = (1 + g_m R'_L) C_{b'e}$$

$$C_{M2} \approx C_{b'e}$$

$$C_{M2} \ll C_{M1}$$

输出回路的时间常数
远小于输入回路时间常数，
考虑高频响应时可以忽略
 C_{M2} 的影响。

$$C = C_{b'e} + C_{M1}$$



6.4.4 单级共射极放大电路的频率响应

1. 高频响应

① Π 型高频等效电路

目标：简化和变换

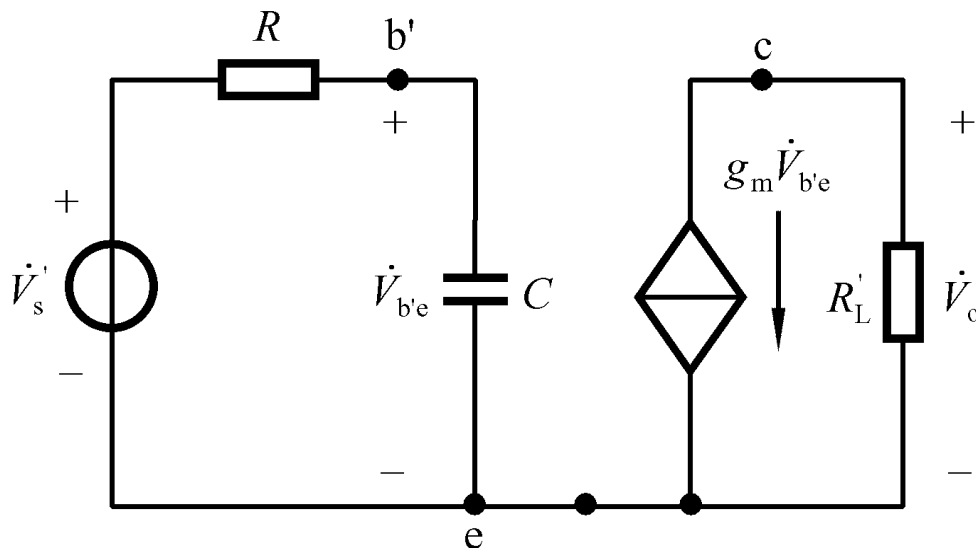
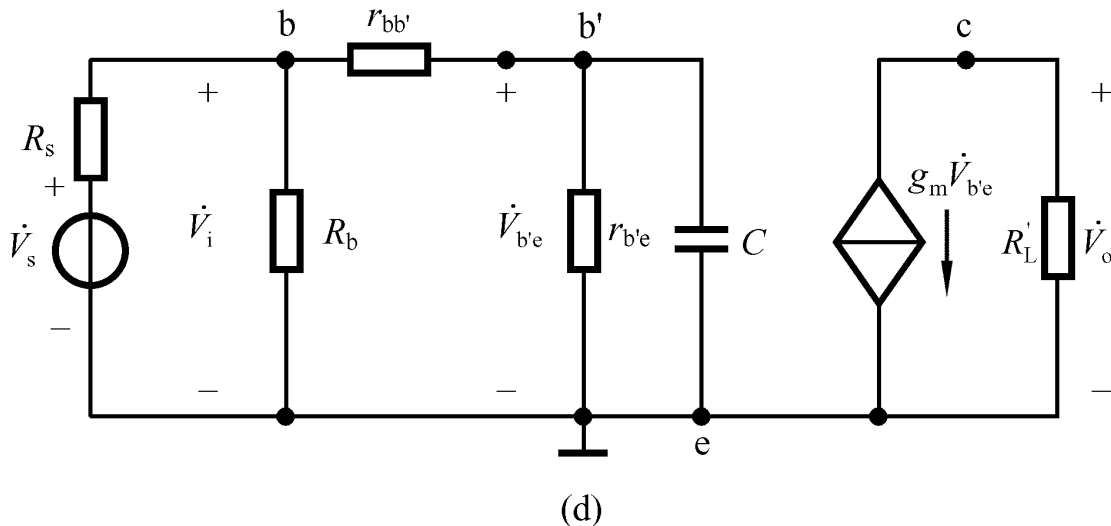
$$C_{M1} = (1 + g_m R'_L) C_{b'c}$$

$$C = C_{b'e} + C_{M1}$$

$$R = (R_s // R_b + r_{bb'}) // r_{b'e}$$

$$\dot{V}'_s = \frac{r_{b'e}}{r_{be}} \cdot \frac{R_b // r_{be}}{R_s + R_b // r_{be}} \dot{V}_s$$

$$r_{be} = r_{bb'} + r_{b'e}$$



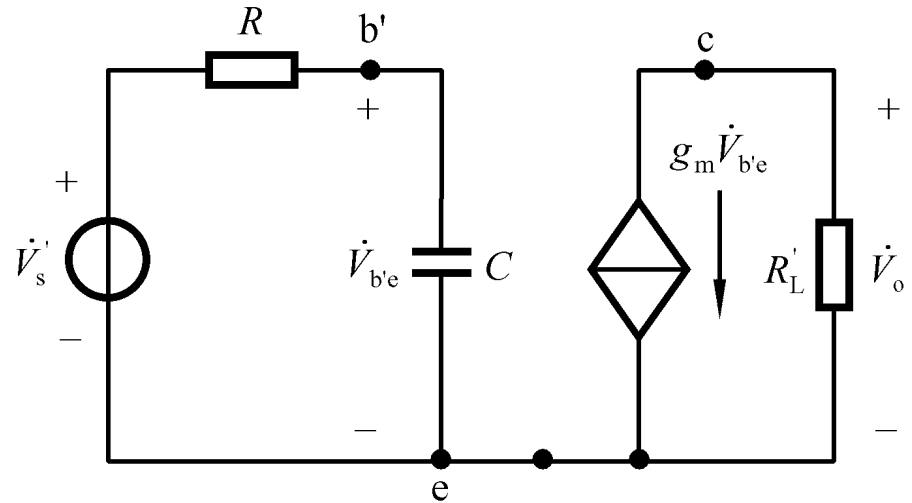
6.4.4 单级共射极放大电路的频率响应

1. 高频响应

② 高频响应和上限频率

由电路得

$$\begin{cases} \dot{V}_{b'e} = \frac{1}{1 + j\omega RC} V'_s \\ \dot{V}_o = -g_m R'_L \dot{V}_{b'e} \\ V'_s = \frac{r_{b'e}}{r_{be}} \cdot \frac{R_b // r_{be}}{R_s + R_b // r_{be}} \dot{V}_s \end{cases}$$



$$C = C_{b'e} + C_{M1} \quad R = (R_s // R_b + r_{bb'}) // r_{b'e}$$

电压增益频响 $\dot{A}_{VSH} = \frac{\dot{V}_o}{\dot{V}_s} = -g_m R'_L \cdot \frac{r_{b'e}}{r_{be}} \cdot \frac{R_b // r_{be}}{R_s + R_b // r_{be}} \cdot \frac{1}{1 + j\omega RC} = \frac{\dot{A}_{VSM}}{1 + j(f/f_H)}$

其中 $\dot{A}_{VSM} = -g_m R'_L \cdot \frac{r_{b'e}}{r_{be}} \cdot \frac{R_b // r_{be}}{R_s + R_b // r_{be}} = -\frac{\beta_0 R'_L}{r_{be}} \cdot \frac{R_b // r_{be}}{R_s + R_b // r_{be}}$ 中频增益或通带源电压增益

$f_H = \frac{1}{2\pi RC}$ 上限频率

1. 高频响应

② 高频响应和上限频率

共射放大电路

$$\dot{A}_{VSH} = \dot{A}_{VSM} \cdot \frac{1}{1 + j(f/f_H)}$$

RC低通电路

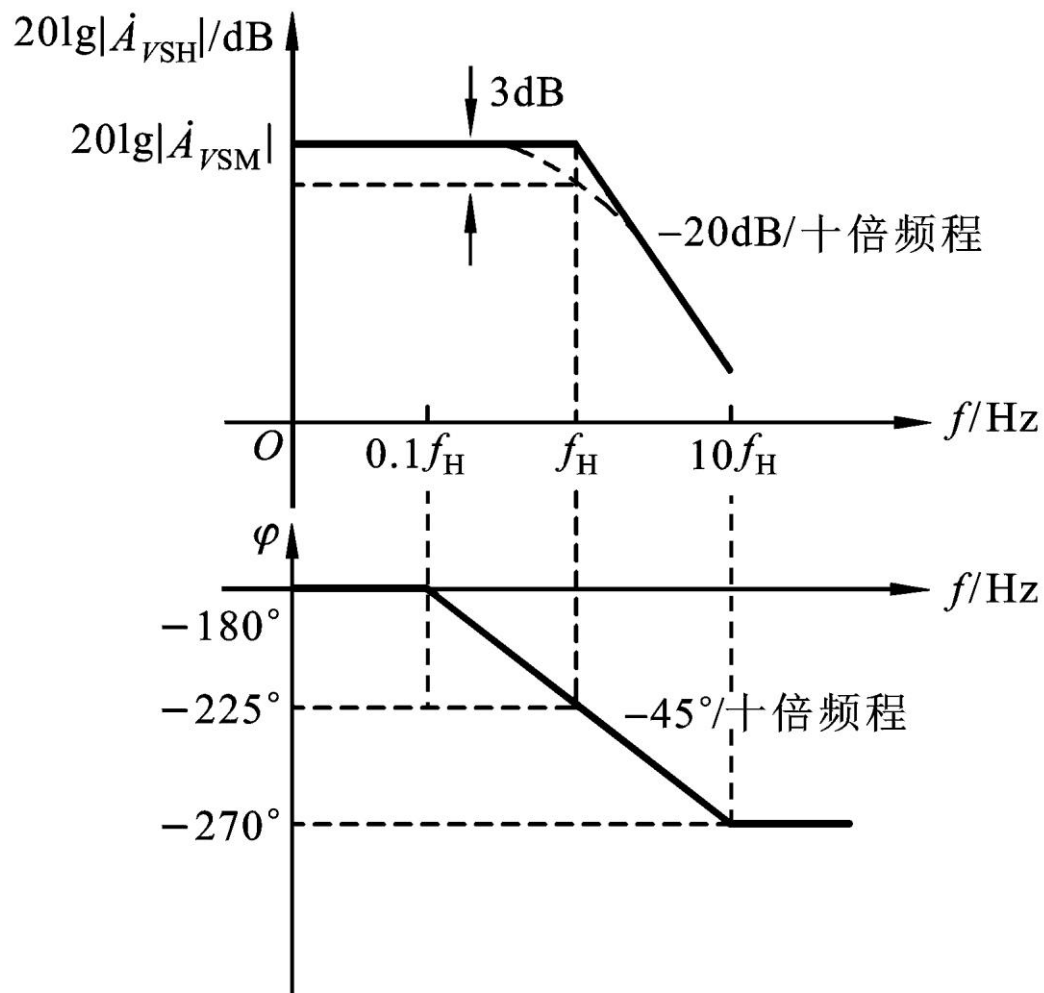
$$\dot{A}_{VH} = \frac{1}{1 + j(f/f_H)}$$

频率响应曲线变化趋势相同

幅频响应

$$20\lg|\dot{A}_{VSH}| = 20\lg|\dot{A}_{VSM}| + 20\lg \frac{1}{\sqrt{1 + (f/f_H)^2}}$$

相频响应 $\varphi = -180^\circ - \arctg(f/f_H)$



例题

例3.7.1 设共射放大电路在室温下运行，其参数为： $R_s = 1\text{k}\Omega$ ， $r_{bb'} = 100\Omega$ ， $I_C = 1\text{mA}$ ， $\beta_0 = 100$ ， $f_T = 400\text{MHz}$ ， $C_{b'c} = 0.5\text{pF}$ ， $R_c = 5\text{k}\Omega$ 。负载开路， R_b 足够大忽略不计。试计算它的低频电压增益和上限频率。

解：模型参数为

$$g_m = \frac{I_E}{V_T} = \frac{1\text{mA}}{26\text{mV}} = 0.038 \text{ S}$$

$$r_{b'e} = \frac{\beta_0}{g_m} = \frac{100}{0.038 \text{ S}} = 2.6 \text{ k}\Omega$$

$$C_{b'e} = \frac{g_m}{2\pi f_T} - C_{b'c} = 14.8 \text{ pF}$$

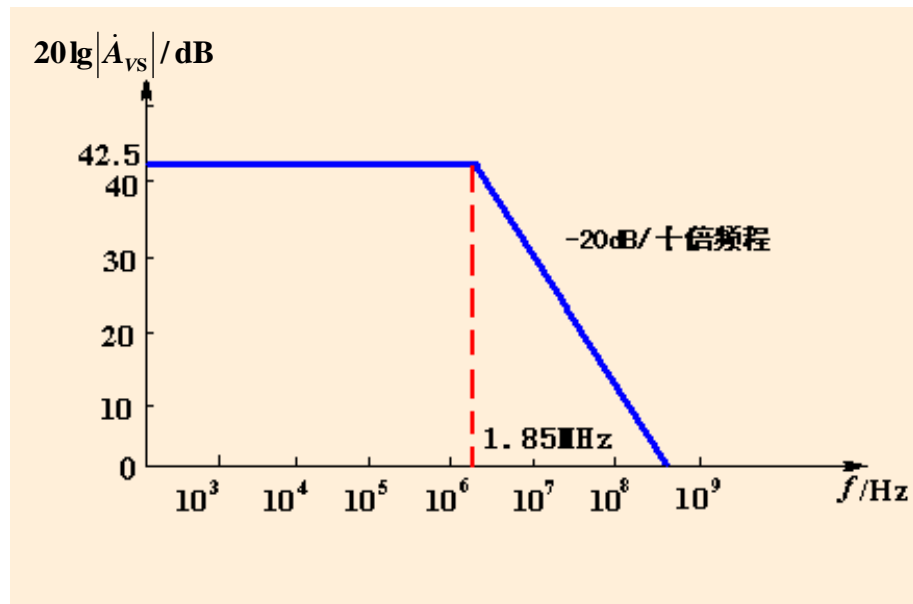
$$C_{M1} = (1 + g_m R_c) C_{b'c} = 96.7 \text{ pF}$$

低频电压增益为

$$\dot{A}_{VSM} = -g_m R_c \frac{r_{b'e}}{R_s + r_{bb'} + r_{b'e}} = -133.51$$

$$20 \lg |\dot{A}_{VSM}| = 20 \lg |-133.51| = 42.5 \text{ dB}$$

又因为 $R = (R_s + r_{bb'}) // r_{b'e} = 0.77 \text{ k}\Omega$



$$C = C_{b'e} + C_{M1} = 111.5 \text{ pF}$$

所以上限频率为

$$f_H = \frac{1}{2\pi RC} = 1.85 \text{ MHz}$$